

Seven-Disk Manifold, α -attractors and B-modes

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Abstract

Cosmological α -attractor models in $\mathcal{N} = 1$ supergravity are based on hyperbolic geometry of a Poincaré disk with the radius square $\mathcal{R}^2 = 3\alpha$. The predictions for the B-modes, $r \approx 3\alpha \frac{4}{N^2}$, depend on moduli space geometry and are robust for a rather general class of potentials. Here we notice that starting with M-theory compactified on a 7-manifold with G_2 holonomy, with a special choice of Betti numbers, one can obtain d=4 $\mathcal{N} = 1$ supergravity with rank 7 scalar coset $\left[\frac{SL(2)}{SO(2)}\right]^7$. In a model where these 7 unit size Poincaré disks have identified moduli one finds that $3\alpha = 7$. Assuming that the moduli space geometry of the phenomenological models is inherited from this version of M-theory, one would predict $r \approx 10^{-2}$ for $N = 53$ e-foldings. We also describe the related maximal supergravity and M/string theory models leading to preferred values $3\alpha = 1, 2, 3, 4, 5, 6, 7$.

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1 Introduction

To compare the predictions of theoretical models with the observational data on inflationary cosmology [1] one has to use some form of d=4 Einstein theory. In particular one can use $\mathcal{N} = 1$ supergravity models making a choice of the Kähler potential and a superpotential to fit the data. Cosmological models called α -attractor models [2, 3, 4], based on hyperbolic geometry of a Poincaré disk with the radius square 3α , are in good agreement with the data. The tilt of the spectrum of fluctuations and the level of B-modes depend on the number of e-foldings N and on the moduli space curvature $\mathcal{R}_{\text{Kähler}} = -\frac{2}{3\alpha}$:

$$n_s \approx 1 - \frac{2}{N}, \quad r \approx 3\alpha \frac{4}{N^2}. \quad (1.1)$$

This prediction is valid for α -attractor models with $\alpha \lesssim O(10)$ for rather general class of potentials described in [2, 3, 4]. The early versions of these models were derived in [2], the more advanced versions were presented in [3, 4]. At the level of phenomenological $\mathcal{N} = 1$ supergravity any value of $0 < 3\alpha < \infty$ is acceptable, so one can view the future detection of the B-modes, or a new bound on r , as an experimental information about the curvature of the moduli space in these phenomenological models.

However, one may try to motivate certain preferred values of the Poincaré disk radius square 3α as originating from a fundamental theory underlying $\mathcal{N} = 1$ supergravity. It was already suggested in [3] that the lowest possible value $3\alpha = 1$, with one unit size Poincaré disk, is motivated by a maximal superconformal $\mathcal{N} = 4$ theory [5] and $\mathcal{N} = 4$ pure supergravity without matter [6].

In this note we will study the possible origin of the moduli space geometries in maximal $\mathcal{N} = 8$ supergravity and M/string theory. We assume that when the maximally supersymmetric theories are reduced to $\mathcal{N} = 1$ phenomenological α -attractor models, some mechanism of generating the required potentials will take place, but the moduli space geometry will be inherited from the more fundamental theories.

In this setting we will find a reasonably well motivated models of the Poincaré disk with radius square 3α taking values 1, 2, 3, 4, 5, 6, 7. In particular, the case with the highest value of $3\alpha = 7$ suggests that r is only slightly below 10^{-2} .

Joint analysis of the data from BICEP2/Keck and Planck experiments [1] yields an upper limit on B-modes, $r \leq 7 \times 10^{-2}$. The new interesting target with preferred values of α originating in M/string theory, for the number of e-foldings $47 < N < 57$, is now

$$3\alpha = 7 : \quad r \approx 7 \frac{4}{N^2}, \quad 0.86 \times 10^{-2} < r < 1.3 \times 10^{-2}, \quad (1.2)$$

and the lowest one in the context of maximal $\mathcal{N} = 4$ superconformal theory is

$$3\alpha = 1 : \quad r \approx \frac{4}{N^2}, \quad 1.2 \times 10^{-3} < r < 1.8 \times 10^{-3}. \quad (1.3)$$

2 Poincaré disk with the radius square 3α

Consider the Kähler potential

$$K = -3\alpha \ln(1 - Z\bar{Z}) . \quad (2.1)$$

It describes a Poincaré disk with the radius square 3α . The metric of the moduli space is $g_{Z\bar{Z}} = K_{Z\bar{Z}} = \frac{3\alpha}{(1-Z\bar{Z})^2}$. The Kähler manifold curvature computed from this metric depends on α :

$$\mathcal{R}_{\text{Kähler}} = -g_{Z\bar{Z}}^{-1} \partial_Z \partial_{\bar{Z}} \log g_{Z\bar{Z}} = -\frac{2}{3\alpha} . \quad (2.2)$$

The kinetic term for the complex scalar field is

$$ds^2 = \frac{3\alpha}{(1 - Z\bar{Z})^2} dZ d\bar{Z} = \frac{dx^2 + dy^2}{\left(1 - \frac{x^2 + y^2}{3\alpha}\right)^2} , \quad (2.3)$$

where $Z = (x + iy)/\sqrt{3\alpha}$. For the vanishing inflaton¹ the kinetic term becomes in terms of the inflaton $Z = \bar{Z} = \tanh \frac{\varphi}{\sqrt{6\alpha}}$

$$3\alpha \frac{\partial_\mu Z \partial^\mu Z}{(1 - Z)^2} = \frac{1}{2} (\partial_\mu \varphi)^2 . \quad (2.4)$$

In these models the potentials depend on a geometric variable $Z = \bar{Z}$

$$V = V\left(\tanh \frac{\varphi}{\sqrt{6\alpha}}\right) . \quad (2.5)$$

2.1 Half-plane variables

One can use an alternative description of the same physical system by making a choice $\frac{1+Z}{1-Z} = -i\tau$.

$$K = -3\alpha \ln(-i(\tau - \bar{\tau})) . \quad (2.6)$$

The kinetic term for the complex scalar field is

$$ds^2 = 3\alpha \frac{d\tau d\bar{\tau}}{(2\text{Im}\tau)^2} . \quad (2.7)$$

In this form the kinetic term has an $SL(2, \mathbb{R})$ symmetry

$$\tau' = \frac{a\tau + b}{c\tau + d}, \quad ad - bc \neq 0 , \quad (2.8)$$

where a, b, c, d are real numbers and

$$\frac{d\tau d\bar{\tau}}{(\tau - \bar{\tau})^2} = \frac{d\tau' d\bar{\tau}'}{(\tau' - \bar{\tau}')^2} . \quad (2.9)$$

When the inflaton $\tau + \bar{\tau}$ vanishes at $\tau = -\bar{\tau} = ie\sqrt{\frac{2}{3\alpha}}\varphi$

$$3\alpha \frac{d\tau d\bar{\tau}}{(2\text{Im}\tau)^2} = \frac{1}{2} (\partial_\mu \varphi)^2 . \quad (2.10)$$

¹The inflaton field $Z - \bar{Z}$ can be either stabilized to become heavy, or in models with constrained orthogonal superfields this field depends on fermions and does not participate in cosmological evolution.

3 Seven-disk geometry in maximal supergravity

Before looking at M-theory on a 7-manifold with G_2 holonomy we will explain the origin of the seven-disk geometry starting from D=4 $\mathcal{N} = 8$ supergravity. M theory/d=11 supergravity can be compactified on a 7-torus, which leads to d=4 maximal $\mathcal{N} = 8$ supergravity [7] upon dualization of the form fields. This model has 70 scalars in the coset space $\frac{E_{7(7)}}{SU(8)}$ and $E_{7(7)}$ symmetry. Following [8], we consider truncation of $\mathcal{N} = 8$ supergravity [7] to $\mathcal{N} = 4$ supergravity interacting with six $\mathcal{N} = 4$ vector multiplets. The $E_{7(7)}$ symmetry is decomposed as follows

$$E_{7(7)} \supset [SL(2)] \times SO(6, 6) , \quad (3.1)$$

The 70 scalars of $\mathcal{N} = 8$ supergravity [7] are first truncated to

$$70 \rightarrow 2 + 36 \quad (3.2)$$

In the next step one takes into account that

$$SO(6, 6) \supset SO(2, 2) \times SO(2, 2) \times SO(2, 2) \quad (3.3)$$

and

$$36 \rightarrow 3 \times 4 \quad (3.4)$$

so that

$$70 \rightarrow 2(1 + 6) = 2 \times 7 = 14 . \quad (3.5)$$

This truncation has a Kähler structure supporting $\mathcal{N} = 1$ supersymmetry. One can identify 7 Poincaré disks due to the decomposition

$$E_{7(7)}(\mathbb{R}) \supset [SL(2, \mathbb{R})]^7 . \quad (3.6)$$

The original kinetic term is reduced to a form with the Kähler potential of the form

$$K = - \sum_{i=1}^7 \ln(-i(\tau_i - \bar{\tau}_i)) \quad (3.7)$$

with 7 pairs of independent scalars and the $[SL(2, \mathbb{R})]^7$ symmetry, a seven-disk manifold. The fact that the disk of the $SL(2)$ commuting with $SO(6, 6)$ has the same Kähler curvature of the other six $SL(2)/SO(2)$ (each separately corresponding to $\alpha = 1/3$) can be understood by string triality arguments [9] and by the underlying special geometry of the N=2 truncation [10].

4 M theory on a 7-manifold with G_2 holonomy

Instead of a compactification on a 7-torus, one can compactify M theory on a 7-manifold with G_2 holonomy. The early investigation of G_2 holonomy in physics was performed in [11], with review of the first 20 years in [12]. One of the most recent application of this compactification

can be found in [13] and, of course, many more studies of M theory on G_2 were performed over the years.

Here we will focus on the model studied in [14, 15], it requires the following choice of the Betti numbers

$$(b_0, b_1, b_2, b_3) = (1, 0, 0, 7) . \quad (4.1)$$

This theory is identified with the maximal rank reduction on the seven torus and leads directly to d=4 $\mathcal{N} = 1$ ‘curious supergravity’ where 7 complex scalars are coordinates of the coset space

$$\left[\frac{SL(2, \mathbb{R})}{SO(2)} \right]^7 . \quad (4.2)$$

The corresponding Kähler potential describing the scalar sector of this theory is the one in eq. (3.7) with 7 pairs of independent scalars and the $[SL(2, \mathbb{R})]^7$ symmetry. This model is one of the ‘Four curious supergravities’ defined in [15]. The other 3 have $\mathcal{N} = 2$, $\mathcal{N} = 4$, $\mathcal{N} = 8$ supersymmetries, we are interested only in $\mathcal{N} = 1$ ‘curious supergravity’. It has the field content defined by Betti numbers : the d=4 fields originating from the d=11 metric g_{MN} are

$$\begin{aligned} g_{\mu\nu} &\rightarrow b_0 = 1 \\ A_\mu &\rightarrow b_1 = 0 \\ \mathcal{A} &\rightarrow b_1 + b_3 = 7 \end{aligned} \quad (4.3)$$

The ones from d=11 gravitino ψ_M are

$$\begin{aligned} \psi_\mu &\rightarrow b_0 + b_1 = 1 \\ \chi &\rightarrow b_2 + b_3 = 7 \end{aligned} \quad (4.4)$$

The ones from the 3-form A_{MNP} are

$$\begin{aligned} A_{\mu\nu\rho} &\rightarrow b_0 = 1 \\ A_{\mu\nu} &\rightarrow b_1 = 0 \\ A_\mu &\rightarrow b_2 = 0 \\ A &\rightarrow b_3 = 7 \end{aligned} \quad (4.5)$$

To summarize, the field content of the M theory compactified on a 7-manifold with G_2 holonomy and Betti numbers (4.1) is a metric, a gravitino and a 3-form (which has no degrees of freedom, but affects trace anomaly)

$$g_{\mu\nu}, \psi_\mu, A_{\mu\nu\rho} \quad (4.6)$$

and 7 scalars, 7 spin 1/2 fields and 7 pseudoscalars

$$\tau_i = \mathcal{A}_i + iA_i, \chi_i . \quad (4.7)$$

The corresponding Kähler geometry is the seven-disk manifold in (3.7).

For generic Betti numbers (b_0, b_1, b_2, b_3) these models are known to have a generalized mirror symmetry, which flips one set of Betti numbers into the other one,

$$(b_0, b_1, b_2, b_3) \rightarrow (b_0, b_1, b_2 - \rho/2, b_3 + \rho/2) \quad (4.8)$$

and $\rho \equiv 7b_0 - 5b_1 + 3b_2 - b_3$ changes the sign. One of the reason the model we describe here was given a name ‘curious supergravity’ is that it has $\rho = 0$, it is a *self-mirror* in the above sense. It also means that it has a vanishing Weyl anomaly $g_{\mu\nu} \langle T^{\mu\nu} \rangle = -\frac{\rho}{24 \times 32 \pi^2} R^{*\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}^* = 0$, the presence of the 3-form $A_{\mu\nu\rho}$ is important for this.

To connect this compactified M theory model to α -attractor geometry we can make a choice that all moduli in our 7 unit radius disks in (3.7) are identified, namely

$$3\alpha = 7 : \quad \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 \equiv \tau . \quad (4.9)$$

We are left with one Poincaré disk of the radius square 7 times larger than the unit radius square.

$$K = - \sum_{i=1}^7 \ln(-i(\tau_i - \bar{\tau}_i)) = -7 \ln(-i(\tau - \bar{\tau})) . \quad (4.10)$$

$$ds^2 = 7 \frac{d\tau d\bar{\tau}}{(2 \operatorname{Im} \tau)^2} . \quad (4.11)$$

The following interpretation of this identification can be suggested: the diagonal components of the internal space metric g_{ij} are taken to be the same in all 7 directions, $g_{ij} \sim \delta_{ij}$, and the 3-form A_{ijk} , which leads to 7 pseudoscalars in d=4, since $b_3 = 7$, is also the same in all directions. An analogous identification was performed in [16], where an early dimensional reduction of superstring theories was studied. The resulting d=4 $\mathcal{N} = 1$ supergravity, neglecting the matter fields C in [16], has the following Kähler manifold:

$$K = - \ln(-i(s - \bar{s})) - 3 \ln(-i(t - \bar{t})) . \quad (4.12)$$

We will show in the next section that using string theory compactification on a product of 3 tori $T_2 \times T_2 \times T_2 \subset T_6$ one can get the seven-disk geometry.

$$\begin{aligned} K = & - \ln(-i(s - \bar{s})) - \ln(-i(t_1 - \bar{t}_1)) - \ln(-i(t_2 - \bar{t}_2)) - \ln(-i(t_3 - \bar{t}_3)) \\ & - \ln(-i(u_1 - \bar{u}_1)) - \ln(-i(u_2 - \bar{u}_2)) - \ln(-i(u_3 - \bar{u}_3)) \end{aligned} \quad (4.13)$$

Thus, the model (4.12) in [16] corresponds to the one in (4.13) under condition that

$$t_1 = t_2 = t_3 = t, \quad u_1 = u_2 = u_3 = \text{const} . \quad (4.14)$$

This means that some fields of higher-dimensional geometry were discarded, for example all u_i fields and the difference between t_i fields. If instead we would impose on (4.13) the condition

$$s = t_1 = t_2 = t_3 = u_1 = u_2 = u_3 = \tau \quad (4.15)$$

we would reproduce the Kähler geometry (4.10) of the single Poincaré disk of the radius square $3\alpha = 7$. In analogous manner we can get other values

$$3\alpha = \{1, 2, 3, 4, 5, 6, 7\} \quad (4.16)$$

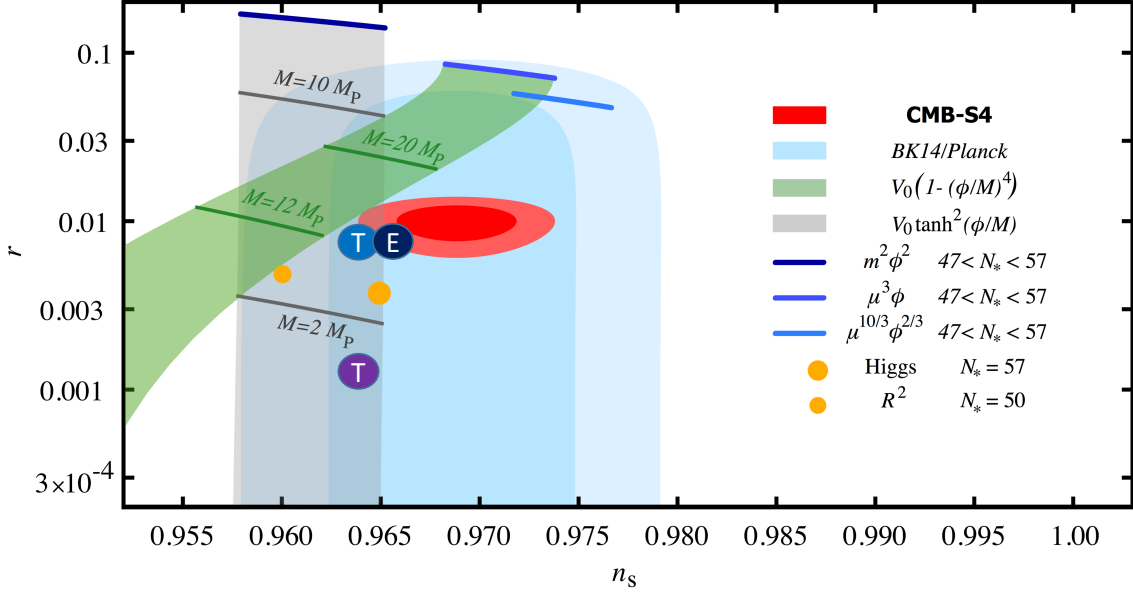


Figure 1: This Figure is taken from [17], it represents a forecast of CMB-S4 constraints in the $n_s - r$ plane for a fiducial model with $r = 0.01$. Here the grey band shows predictions of the sub-class of α -attractor models [2, 3, 4]. We have added to this figure a blue circle with the letter T inside it corresponding to a highest preferred value $3\alpha = 7$ and the purple one corresponding to the lowest preferred value $3\alpha = 1$ in a seven-disk geometry. All intermediate cases $3\alpha = \{1, 2, 3, 4, 5, 6, 7\}$ are between these two. They all describe the class of α -attractor models with $V \sim \tanh^2(\varphi/\sqrt{6\alpha})$, so-called quadratic T -models. The quadratic E -models with $V \sim (1 - e^{\sqrt{2/3}\alpha\varphi})^2$ tend to be slightly to the right of the T -models, see [2]. We show them as a navy circle with the letter E inside it.

by requiring that

$$\begin{aligned}
 3\alpha = 7 : \quad & \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 \equiv \tau \\
 3\alpha = 6 : \quad & \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 \equiv \tau, \quad \tau_7 = \text{const} \\
 3\alpha = 5 : \quad & \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 \equiv \tau, \quad \tau_6 = \tau_7 = \text{const} \\
 3\alpha = 4 : \quad & \tau_1 = \tau_2 = \tau_3 = \tau_4 \equiv \tau, \quad \tau_5 = \tau_6 = \tau_7 = \text{const} \\
 3\alpha = 3 : \quad & \tau_1 = \tau_2 = \tau_3 \equiv \tau, \quad \tau_4 = \tau_5 = \tau_6 = \tau_7 = \text{const} \\
 3\alpha = 2 : \quad & \tau_1 = \tau_2 \equiv \tau, \quad \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \text{const} \\
 3\alpha = 1 : \quad & \tau_1 \equiv \tau, \quad \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \text{const}
 \end{aligned} \tag{4.17}$$

We illustrate in Fig. 1 the features of α -attractor models [2, 3, 4] with the seven-disk geometry using the recent discussion of B-modes in the CMB-S4 Science Book [17]. We show in Fig. 1 predictions of α -attractor models with seven-disk geometry in the $n_s - r$ plane for $N \sim 55$, for the minimal value $3\alpha = 1$ and for the maximal value $3\alpha = 7$.

5 Values of 3α in string theory

Here we will show how to derive the 7-disk geometry (4.13) in string theory. We start with the derivation of non-compact symmetries in string theory following [18], [19]. The toroidal

compactification to d=4 of the $\mathcal{N} = 1$ supergravity/string theory in d=10 space-time leads to scalars in $\frac{SO(6,6)}{SO(6) \times SO(6)}$ coset space² upon truncation of non-geometric moduli from the d=10 vector multiplets.

As the result of the dimensional reduction one finds a d=4 action for the scalars of the following form,

$$\int d^4x \sqrt{-g} e^{-\phi} (\mathcal{L}_1 + \mathcal{L}_2) . \quad (5.1)$$

Here

$$\mathcal{L}_1 = R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} , \quad (5.2)$$

and

$$\mathcal{L}_2 = \frac{1}{8} \text{tr}(\partial_\mu M^{-1} \partial^\mu M) . \quad (5.3)$$

Here M is a symmetric $O(6,6)$ matrix

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} , \quad (5.4)$$

where $G_{\alpha\beta}$ and $B_{\alpha\beta}$ are the internal space metric and a 2-form, $\alpha, \beta = 1, \dots, 6$. Together they represent the 36 coordinates of the coset space $\frac{SO(6,6)}{SO(6) \times SO(6)}$, we recover the moduli space of the six-torus T_6 in string theory. We would like now to perform the truncation of the 6-torus to three T_2 so that

$$T_2 \times T_2 \times T_2 \subset T_6 \quad (5.5)$$

This corresponds to the reduction $SO(6,6) \supset [SO(2,2)]^3$ and analogous reduction on the coset representative

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \rightarrow \left[\frac{SO(2,2)}{SO(2) \times SO(2)} \right]^3 . \quad (5.6)$$

This means that we keep the following 9 components of $G_{\alpha\beta}$

$$G_{(IJ)} = (g_{11}, g_{22}, g_{12}; g_{33}, g_{44}, g_{34}; g_{55}, g_{66}, g_{56}) , \quad (5.7)$$

and 3 components of $B_{\alpha\beta}$

$$B_{[IJ]} = (b_{12} \equiv b_1, b_{34} \equiv b_2, b_{56} \equiv b_3) . \quad (5.8)$$

We also introduce notation

$$g_1 \equiv g_{11}g_{22} - g_{12}^2 , \quad g_2 \equiv g_{33}g_{44} - g_{34}^2 , \quad g_3 \equiv g_{55}g_{66} - g_{56}^2 . \quad (5.9)$$

Now we observe that the coset $\frac{SO(2,2)}{SO(2) \times SO(2)}$ is isomorphic to $\frac{SL(2, \mathbb{R})}{SO(2)} \times \frac{SL(2, \mathbb{R})}{SO(2)}$ and so we can package the $SO(2,2)$ matrix into an $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ matrix. We will do this for all three

²In general, in the case of the heterotic string theory one finds scalars in the $\frac{SO(6,6+n)}{SO(6) \times SO(6+n)}$ coset space. Here the scalars in the $\frac{SO(6,6)}{SO(6) \times SO(6)}$ part of the coset space originate from the geometric moduli, whereas the additional ones with $n \neq 0$ originate from the matter vector multiplets in d=10. If we keep some of the vector multiplets, so that $n > 0$ we do not find models with Poincaré disk geometry.

copies of $\frac{SO(2,2)}{SO(2) \times SO(2)}$ cosets, following an example of one of them in [19]. We have 4 real scalars from $g_{11}, g_{22}, g_{12}, b_{12}$. We package them as follows: $t_1 \equiv b_1 + i\sqrt{g_1}$ and $u_1 \equiv \frac{g_{12}}{g_{22}} + i\frac{\sqrt{g_1}}{g_{22}}$. The inverse relation is for the 2x2 matrices

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \frac{\text{Im } t_1}{\text{Im } u_1} \begin{pmatrix} u_1 u_1^* & \text{Re } u_1 \\ \text{Re } u_1 & 1 \end{pmatrix} \quad (5.10)$$

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix} = \text{Re } t_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (5.11)$$

In the same way we can organize all 6 complex scalars, 3 of them are often called Kähler moduli

$$t_1 = b_1 + i\sqrt{g_1}, \quad t_2 = b_2 + i\sqrt{g_2}, \quad t_3 = b_3 + i\sqrt{g_3}, \quad (5.12)$$

and the other 3 are called complex structure moduli

$$u_1 = \frac{g_{12}}{g_{22}} + i\frac{\sqrt{g_1}}{g_{22}}, \quad u_2 = \frac{g_{34}}{g_{44}} + i\frac{\sqrt{g_2}}{g_{44}}, \quad u_3 = \frac{g_{56}}{g_{66}} + i\frac{\sqrt{g_3}}{g_{66}}. \quad (5.13)$$

This corresponds to reorganizing $\left[\frac{SO(2,2)}{SO(2) \times SO(2)}\right]^3$ into $\left[\frac{SL(2, \mathbb{R})}{SO(2)}\right]^6$. The corresponding Kähler potentials are $K(t_i, \bar{t}_i) = -\ln(-i(t_i - \bar{t}_i))$ and $K(u_i, \bar{u}_i) = -\ln(-i(u_i - \bar{u}_i))$.

One more important step here is to switch from the string frame as in (5.1) to the Einstein frame in d=4, which is a well known procedure of rescaling the metric, see for example [16]. As the result, we find an action with $\mathcal{N} = 1$ supersymmetry and 7 complex scalars. The axion, dual to $H_{\mu\nu\lambda}$, and dilaton as shown in eq. (5.2) form a complex scalar

$$s = a + ie^\phi \quad (5.14)$$

with the Kähler potential $K = -\ln(-i(s - \bar{s}))$. The complete Kähler potential of the string theory dimensionally reduced on $T_2 \times T_2 \times T_2 \subset T_6$ is now given by the expression in (4.13) in the previous section, as promised there.

Thus here again we reproduced the 7 Poincaré disk geometry of the unit radius each. We may now study the same cases as we did in the previous section: the conclusion is as in M-theory compactified on 7-manifold with G_2 holonomy in eq. (4.16) which gives us seven possible values of r , according to (1.1), for example for $N = 55$

$$r \approx \{1.3, 2.6, 3.9, 5.2, 6.5, 7.8, 9.1\} \times 10^{-3} \quad (5.15)$$

6 Conclusion

In conclusion, we made an assumption that the moduli space geometry of the phenomenological $\mathcal{N} = 1$ α -attractor models in [2, 3, 4] is inherited from the M-theory compactified on 7-manifold with G_2 holonomy to a ‘curious $\mathcal{N} = 1$ supergravity’ [15], or from truncated $\mathcal{N} = 8$ maximal supergravity, or from toroidally compactified string theory. In such case we argued that the

possible cosmological α -attractor models might come with the values of $3\alpha = 1, 2, 3, 4, 5, 6, 7$ when some of the higher dimensional fields are discarded, following the procedure employed in the past in [16] and presented in eq. (4.17). To make a step from preferred values for 3α to a realistic prediction we would need to find the origin of the suitable class of potentials in these theories.

The relevant preferred values of the ratio of the tensor to scalar fluctuations during inflation are shown in eq. (5.15). We illustrated the position of these models in $n_s - r$ plane in Fig. 1. The highest one $r \approx 10^{-2}$ will be the first interesting target for the B-mode experiments as well as for the theoretical studies of realistic cosmological models based on the seven-disk geometry.

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